

Quantum effects to the Higgs boson self-couplings in the SM and in the MSSM

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Abstract

We show that the effects of heavy Higgs particles and heavy top-squarks in the one-loop self-couplings of the lightest CP-even MSSM Higgs boson decouple from the low energy theory when the self-couplings are expressed in terms of the Higgs boson mass M_{h^0} . Our conclusion is that the h^0 self-interactions become very close to those of the SM Higgs boson and, therefore, MSSM quantum effects could only be revealed by very high precision experiments.

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1 Introduction

One of the most important issues at the next generation of colliders is the discovery of at least one light Higgs boson particle and the elucidation of the mechanism of symmetry breaking [1]. Particularly relevant in order to establish the Higgs mechanism experimentally in an unambiguous way, is the reconstruction of the Higgs self-interaction potential. This task requires the measurement of the trilinear and quartic self-couplings as predicted in the Standard Model (SM) or in supersymmetric (SUSY) theories. Since the predictions of these self-couplings are different in both theories, their experimental measurement could provide not just an essential way to determine the mechanism for generating the masses of the fundamental particles but also an indirect way to test SUSY. The cross section for double Higgs production (e.g., Zhh) is related to the triple Higgs self-couplings λ_{hhh} , which in turn is related to the spontaneous symmetry breaking shape of the Higgs potential. Experimental studies indicate that for a SM-like Higgs boson with $m_h = 120$ GeV at $1000 fb^{-1}$, a precision of $\delta\lambda_{hhh}/\lambda_{hhh} = 23\%$ is possible at TESLA [3]. Strategies for measuring the SM Higgs boson self-couplings at the LHC have been also discussed recently in [4]. Many other studies have addressed the issue of the measurement of the neutral Higgs self-couplings in the Minimal Supersymmetric Standard Model (MSSM) and also in the two-Higgs doublet model (2HDM) [5, 6].

In recent papers, we investigated how far the MSSM Higgs potential reproduces the SM potential when the non-standard particles are heavy [7, 8]. More concretely, we explore the decoupling behaviour of the radiative corrections to the h^0 self-couplings at the one-loop level,

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both numerically and analytically. A first step into this direction was the analysis of the leading Yukawa contributions of order $\mathcal{O}(m_t^4)$ to the lightest MSSM Higgs boson self-couplings [7]. Recently, an analytical study of the contributions from the MSSM Higgs sector itself [8] has been done. This talk gives a summary on the results of these calculations.

First, we present in Table 1 the trilinear and quartic SM and MSSM Higgs boson self-couplings, at the tree-level. Clearly, for arbitrary values of the MSSM Higgs-sector input parameters, $\tan\beta$ and M_{A^0} , the values of the h^0 self-couplings are different from those of the SM Higgs boson. However, the situation changes in the so-called *decoupling limit* of the Higgs sector where $M_{A^0} \gg M_Z$, yielding a particular spectrum with heavy H^0, H^\pm, A^0 Higgs bosons with degenerate masses, and a light h^0 boson having a tree-level mass of $m_{h^0} \simeq M_Z |\cos 2\beta|$ [9]. In this limit, which also implies $\alpha \rightarrow \beta - \pi/2$, the h^0 self-couplings converge respectively to $\lambda_{hhh}^0 \simeq 3g/2 M_W M_{h^0}^{2\text{tree}}$ and $\lambda_{hhhh}^0 \simeq 3g^2/4 M_W^2 M_{h^0}^{2\text{tree}}$ and therefore, they converge to their respective SM Higgs boson self-couplings with the same mass. Thus, we can conclude that, at the tree level, there is decoupling of the heavy MSSM Higgs sector in the $M_{A^0} \gg M_Z$ limit. Therefore, by studying the light Higgs boson self-interactions it will be very difficult to unravel its SUSY origin. This is another reason to investigate the effects of the quantum contributions to these self-interactions.

ϕ		$\lambda_{\phi\phi\phi}$	$\lambda_{\phi\phi\phi\phi}$
SM	H	$\frac{3gM_H^2}{2M_W}$	$\frac{3g^2M_H^2}{4M_W^2}$
MSSM	h^0	$\frac{3gM_Z}{2c_W} \cos 2\alpha \sin(\beta + \alpha)$	$\frac{3g^2}{4c_W^2} \cos^2 2\alpha$

Table 1: Tree-level SM and MSSM Higgs boson self-couplings

2 One-loop Higgs sector contributions in the *decoupling limit*

We summarize here the one-loop results for the h^0 self-couplings from the MSSM Higgs sector itself by considering the already described *decoupling limit* [8]. We only consider the set of diagrams that provides contributions to the MSSM h^0 self-couplings different from the SM ones. We have checked explicitly that the one-loop contributions from diagrams that have at least one gauge boson flowing in the loops are the same in both models. Consequently, we are considering, first, diagrams involving only the MSSM heavy Higgs bosons (H^0, H^\pm , and A^0), second, diagrams with these heavy particles and the Goldstone bosons or the lightest Higgs boson appearing in the same loop, and finally, diagrams involving just Goldstone bosons and the lightest Higgs boson in the loops (*light* contributions); all of them are, in principle, different in the two models. However, as explained in detail in [8], we found that the one-loop *light* contributions approach the SM ones in the $M_{A^0} \gg M_Z$ limit and therefore, they become indistinguishable in both models. This is equivalent to saying that the difference between the one-loop unrenormalized vertex functions of the two theories in the *decoupling limit* is coming from diagrams including at least one heavy MSSM Higgs particle.

The results for the one-loop contributions to the n -point vertex functions, $\Delta\Gamma_{h^0}^{(n)}$, coming from the Higgs sector itself, can be summarized generically as

$$\begin{aligned} \Delta\Gamma_{h^0}^{(n)} \simeq & M_Z^2 \left[\mathcal{O}\left(\frac{1}{\epsilon}\right) + \mathcal{O}\left(\log \frac{M_{EW}^2}{\mu_0^2}\right) + \mathcal{O}\left(\log \frac{M_{A^0}^2}{\mu_0^2}\right) + \text{finite terms} \right] \\ & + M_{A^0}^2 \left[\mathcal{O}\left(\frac{1}{\epsilon}\right) + \mathcal{O}\left(\log \frac{M_{A^0}^2}{\mu_0^2}\right) + \text{finite terms} \right], \end{aligned} \quad (1)$$

where $M_{EW}^2 \equiv M_Z^2, M_W^2, m_{h^0}^2$. These contributions are UV-divergent and contain both a logarithmic dependence and a quadratic dependence on the heavy pseudoscalar mass M_{A^0} . Therefore, all potential non-decoupling effects of the heavy MSSM Higgs particles manifest themselves as divergent contributions in $D = 4$ and some finite contributions, logarithmically and quadratically dependent on M_{A^0} .

As a next step, renormalization has to be done to get finite vertex functions and physical observables; this is performed in practice by adding appropriate counterterms. The results of the vertex counterterms in the *decoupling limit* as well as the results for the renormalization constants, obtained by using the on-shell scheme [10], have been presented in [8]. The results for the renormalized vertex functions, $\Delta\Gamma_{R h^0}^{(n)}$, defined by summing $\Delta\Gamma_{h^0}^{(n)}$ and the counterterm contributions, can be expressed as follows,

$$\begin{aligned}\Delta\Gamma_{R h^0}^{(2)} &= \Delta M_{h^0}^2, \\ \Delta\Gamma_{R h^0}^{(3)} &= \frac{3g}{2M_Z c_W} \Delta M_{h^0}^2 + \frac{g^3}{64\pi^2 c_W^3} M_Z \cos^2 2\beta \Psi_{\text{MSSM}}^{\text{rem}}, \\ \Delta\Gamma_{R h^0}^{(4)} &= \frac{3g^2}{4M_Z^2 c_W^2} \Delta M_{h^0}^2 + \frac{g^4}{64\pi^2 c_W^4} \cos^2 2\beta \Psi_{\text{MSSM}}^{\text{rem}},\end{aligned}\tag{2}$$

where $\Delta M_{h^0}^2$ represents the h^0 mass-squared correction for the h^0 ,

$$\Delta M_{h^0}^2 = M_Z^2 \left[\mathcal{O}\left(\frac{1}{\epsilon}\right) + \mathcal{O}\left(\log \frac{M_{EW}^2}{\mu_0^2}\right) + \mathcal{O}\left(\log \frac{M_{A^0}^2}{\mu_0^2}\right) + \text{finite terms} \right],\tag{3}$$

and $\Psi_{\text{MSSM}}^{\text{rem}}$ refers to the remaining finite terms resulting exclusively from the *light* contributions. For the discussion, we have dropped those terms which are identical in the SM and the MSSM, i.e. the pure gauge part including the pure Goldstone contributions. As a consequence of considering not a complete set of one-loop diagrams, the mass correction $\Delta M_{h^0}^2$ is UV-divergent. In order to cancel this residual divergence in the renormalized two-point function, it is necessary to include also the diagrams dropped here.

The mass correction (3) contains finite terms proportional to M_Z^2 and logarithmic dependences on the heavy mass M_{A^0} as well as on the electroweak masses. The quadratic heavy-mass terms $\sim M_{A^0}^2$, however, appearing in the unrenormalized vertex functions (1), disappear in the on-shell renormalization procedure. Once the tree-level Higgs-boson mass is replaced by the corresponding one-loop mass, $M_{h^0}^2 = M_{h^0}^{\text{tree}^2} + \Delta M_{h^0}^2$, with $\Delta M_{h^0}^2$ given in (3), we obtain that the singular $\mathcal{O}(1/\epsilon)$ terms and the logarithmic heavy-mass terms also disappear in the renormalized 3- and 4-point functions. Hence, we have an analytic demonstration that the heavy MSSM Higgs bosons decouple in the $M_{A^0} \gg M_Z$ limit.

Nevertheless, after the previously commented terms in the radiative corrections to the trilinear and quartic h^0 self-couplings have been absorbed in the h^0 mass redefinition, other finite terms, contained in $\Psi_{\text{MSSM}}^{\text{rem}}$ in (2), still do remain and could give rise to differences between the predictions of the MSSM and the SM. For the interpretation of these remaining terms it is thus crucial to perform the corresponding one-loop analysis for the self-interactions of the Higgs boson in the SM as well. After renormalization of the trilinear and quartic self-couplings in the SM, done in [8], we find

$$\Delta\Gamma_{R H_{SM}}^{(3)} = \frac{g^3}{64\pi^2 c_W^3} M_Z \Psi_{SM}^{\text{rem}}, \quad \Delta\Gamma_{R H_{SM}}^{(4)} = \frac{g^4}{64\pi^2 c_W^4} \Psi_{SM}^{\text{rem}},\tag{4}$$

with the $M_{H_{SM}}$ -dependent function Ψ_{SM}^{rem} representing the only remaining finite terms in the renormalized H_{SM} self-couplings. In general, these finite contributions are different from the

finite $\Psi_{\text{MSSM}}^{\text{rem}}$ terms originating from the *light* contributions in the MSSM. However, by identifying $M_{h^0}^{\text{tree}^2} \simeq M_Z^2 \cos^2 2\beta \longleftrightarrow M_{H_{\text{SM}}}^2$ in the *decoupling limit*, we obtain the asymptotic relation $\Psi_{\text{MSSM}}^{\text{rem}} \longrightarrow \Psi_{\text{SM}}^{\text{rem}}$. Therefore, these EW-finite terms are common to both the lightest MSSM Higgs boson, h^0 , and the SM H_{SM} , in the case $M_{A^0} \gg M_Z$.

Thus, we have shown that the MSSM *heavy* Higgs one-loop contributions can be absorbed in the redefinition of the lightest Higgs boson mass M_{h^0} and therefore, decoupling of the heavy Higgs particles occurs. Similarly, the divergent part of the *light* contributions as well as part of the finite terms are also absorbed in this M_{h^0} mass correction. Another part of the finite terms in the renormalized 3- and 4-point MSSM vertex functions (2) remains. These remaining contributions, however, coincide with the corresponding SM ones in the $M_{A^0} \gg M_Z$ limit by identifying $M_{H_{\text{SM}}}$ with M_{h^0} and, therefore, they drop out when differences in the predictions of both models are considered. Consequently, the trilinear and quartic h^0 self-couplings at the one-loop level and in the $M_{A^0} \gg M_Z$ limit have the same structure as the SM self-couplings [8].

3 $\mathcal{O}(m_t^4)$ one-loop contributions

The one-loop leading Yukawa corrections from top and stop loop contributions to the renormalized h^0 vertex functions were derived in [7] by considering the *decoupling limit* and a heavy top-squark sector, with \tilde{t} masses large as compared to the electroweak scale. Two different scenarios have been addressed: First, in an analytical study, the two stop masses are heavy but close to each other, i.e. $|m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2| \ll |m_{\tilde{t}_1}^2 + m_{\tilde{t}_2}^2|$ [11]. Second, the other possible scenario where the stop mass splitting is of the order of the SUSY mass scale, which corresponds to $|m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2| \simeq |m_{\tilde{t}_1}^2 + m_{\tilde{t}_2}^2|$, in a numerical analysis.

In the analytical studies, we found that the one-loop contribution to the renormalized two-point function is given by $\Delta\Gamma_{R h^0}^{t,\tilde{t}(2)} = \Delta M_{h^0}^2$, with $\Delta M_{h^0}^2$ being the (leading) one-loop correction to the h^0 mass-squared,

$$\Delta M_{h^0}^2 = -\frac{3}{8\pi^2} \frac{g^2}{M_W^2} m_t^4 \log \frac{m_t^2}{m_{\tilde{t}_1} m_{\tilde{t}_2}}. \quad (5)$$

Thus, we were able to write the results for the trilinear and quartic self-couplings, obtained as the corresponding renormalized vertex functions, in the following way,

$$\begin{aligned} \Delta\Gamma_{R h^0}^{t,\tilde{t}(3)} &= \frac{3}{v} \Delta M_{h^0}^2 - \frac{3}{8\pi^2} \frac{g^3}{M_W^3} m_t^4, \\ \Delta\Gamma_{R h^0}^{t,\tilde{t}(4)} &= \frac{3}{v^2} \Delta M_{h^0}^2 - \frac{3}{4\pi^2} \frac{g^4}{M_W^4} m_t^4. \end{aligned} \quad (6)$$

The UV-divergences are canceled by the renormalization procedure, and the logarithmic heavy mass term, which looks like a non-decoupling effect of the heavy particles in the renormalized vertices, disappears when the vertices are expressed in terms of the Higgs-boson mass shift (5). Therefore, they do not appear directly in related observables, i.e. they decouple. However, some non-logarithmic terms $\mathcal{O}(m_t^4)$ remain in the trilinear and quartic h^0 self-couplings (6). Without these top-mass terms, the self-couplings at the one-loop level have the same form as the tree-level couplings, with the tree-level Higgs-boson mass replaced by the corresponding one-loop mass, $M_{h^0}^2 = M_{h^0}^{\text{tree}^2} + \Delta M_{h^0}^2$.

By deriving the equivalent one-loop $\mathcal{O}(m_t^4)$ contributions in the SM we found that, after on-shell renormalization of the trilinear and quartic couplings in the SM, the results correspond precisely to the two non-logarithmic terms in (6). Hence, these top-mass terms are common to

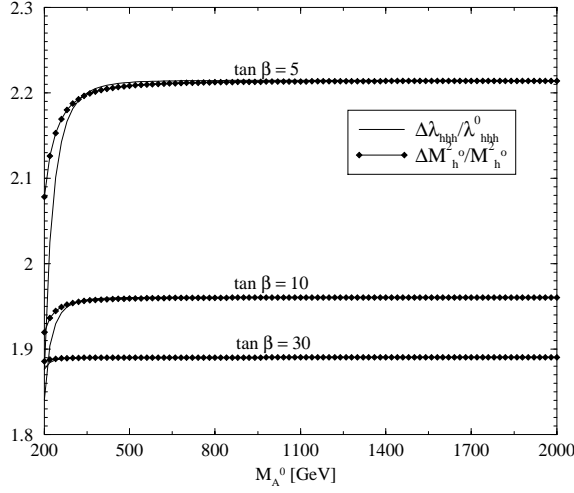


Figure 1: $\mathcal{O}(m_t^4)$ radiative corrections to the trilinear h^0 self-coupling and to the h^0 mass as a function of M_{A^0} , when the stop mass splitting is of $\mathcal{O}(M_{\tilde{Q}})$.

both h^0 and H_{SM} . Therefore, we conclude that the $\mathcal{O}(m_t^4)$ one-loop contributions to the MSSM h^0 vertices *either* represent a shift in the h^0 mass and in the h^0 triple and quartic self-couplings, which can be absorbed in M_{h^0} , *or* reproduce the SM top-loop corrections. The triple and quartic h^0 couplings thereby acquire the structure of the SM Higgs-boson self-couplings. These results have been confirmed also numerically in [7].

The other scenario where the stop-mass splitting is of the order of the SUSY mass scale, has been analyzed numerically, based on the exact results for $\mathcal{O}(m_t^4)$ corrections to the triple and quartic self-couplings. Details of this analysis can be found in [7]. Here we present in Fig. 1 one example of the numerical results for the variation of the trilinear coupling and for the $\mathcal{O}(m_t^4)$ h^0 mass correction as functions of M_{A^0} , for different values of $\tan\beta$, by choosing the set of SUSY input parameters to be $M_{\tilde{Q}} \sim 15$ TeV, $M_{\tilde{U}} \sim \mu \sim |A_t| \sim 1.5$ TeV, such that one gets $|m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2|/|m_{\tilde{t}_1}^2 + m_{\tilde{t}_2}^2| \simeq 0.97$. The radiative correction to the angle α is also taken into account and the non-logarithmic finite contribution to the three-point function owing to the top-triangle diagrams is not taken into account in the figure since, as we mentioned before, it converges always to the SM term. We see in this figure that for very large SUSY scales, the relation $\Delta\lambda_{hhh}/\lambda_{hhh}^0 \approx \Delta M_{h^0}^2/M_{h^0}^{2\text{tree}}$ is again fulfilled. Quantitatively, one finds that for $\tan\beta = 5$ and $M_{A^0} = 2$ TeV, the difference between vertex and mass corrections is $\sim 0.03\%$, and for the most unfavorable case, i.e $\tan\beta = 5$ and $M_{A^0} = 200$ GeV, it is about $\sim 0.2\%$.

Therefore, from the numerical analysis one can conclude that also for the case of a heavy stop system with large mass splitting, the $\mathcal{O}(m_t^4)$ corrections to the trilinear h^0 self-couplings are absorbed to the largest extent in the loop-induced shift of the h^0 mass, leaving only a very small difference, which can be interpreted as the genuine one-loop correction when λ_{hhh} is expressed in terms of M_{h^0} . Similar results have been obtained also for the quartic h^0 self-coupling.

4 Conclusions

We showed that the one-loop Higgs-sector corrections and the $\mathcal{O}(m_t^4)$ Yukawa contributions to the lightest MSSM Higgs-boson self-couplings disappear to a large extent when the self-couplings are expressed in terms of the h^0 -boson mass, in the limit of large M_{A^0} and heavy top squarks, leaving behind the quantum corrections of the SM. Therefore, the triple and quartic h^0 self-couplings acquire the structure of the SM Higgs-boson self-couplings.

Equivalently, we have demonstrated that heavy Higgs particles and heavy top-squarks decouple from the low energy, at the electroweak scale and at one-loop level, and the SM Higgs sector is recovered also in the Higgs self-interactions. Consequently, we would need high-precision experiments for an experimental verification of the supersymmetric nature of the Higgs-boson self-interactions.

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